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# Sensitive Faraday rotation measurement with auto-balanced photodetection

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A magneto-optic polarimetry based on auto-balanced photodetection is investigated. In this experiment, a commercial auto-balanced photoreceiver is adopted to measure the Faraday rotation of air. With a proper setup to utilize its noise cancellation capability, the measurement can be flexible and sensitive. The angular sensitivity is  $2.99 \times 10^{-8} \text{ rad Hz}^{-1/2}$ , which is about 2.7 times the shot noise limit. The measured Verdet constant of air is  $+1.39 \times 10^{-9} \text{ rad G}^{-1} \text{ cm}^{-1}$  at 634.8 nm. Significantly we applied a small AC current to induce the magnetic field, so there was no heating in the coil. In addition, a double current modulation scheme was used to demonstrate that there was no zero drift and amplifier instability in the measurement. The possibility of improvement of the angular sensitivity and the potential applications are also discussed. © 2011 American Institute of Physics. [doi:10.1063/1.3602927]

## I. INTRODUCTION

When a light beam propagates through a material medium on which a magnetic field parallel to the direction of the light is applied, the plane of polarization rotates. This is the Faraday effect discovered by Michael Faraday in 1845.<sup>1</sup> The angle of rotation  $\Delta\theta$  is proportional to both the magnetic induction  $B$  and the optical path length  $d$ , i.e.,  $\Delta\theta = V B d$ , where the Verdet constant,  $V$ , is the characteristic rotation of the material per unit field strength and per unit length. There are a number of practical applications of the Faraday effect measurement so that this magneto-optic polarimetry has continued to attract attention. Highly sensitive magneto-optic polarimetry can be applied to sense very weak magnetic fields,<sup>2</sup> or to measure the Faraday rotation of thin films and materials with low Verdet constant.<sup>3,4</sup>

The rotation of the plane of polarization of linearly polarized light can be viewed as a change in the amplitudes of two orthogonal linear polarized components. A basic approach of the magneto-optic polarimetry is based on measuring the intensity change of one linearly polarized component by means of an analyzer. From Malus's law, we know the intensity of transmitted light is  $I(\theta) = I_0 \cos^2 \theta$ , where  $I_0$  is the intensity of the linearly polarized light incident on the analyzer, and  $\theta$  is the angle between the plane of polarization of the incident light and the polarizing direction of the analyzer. The transmitted light impinges on a photodetector so that the variation in  $I(\theta)$  can be sensed when  $\theta$  is changed.

For improving the detection sensitivity, a balanced photodetection is frequently used.<sup>5</sup> In this setup, a linearly polarized beam to be analyzed is incident on a polarizing beam-splitter (PBS). The two orthogonal components are separated and fall on two photodetectors independently. The difference between the two photocurrents is then taken as a quantity

which will be proportional to the angle of polarization rotation. As the azimuth of polarization of the light relative to the polarizing directions of the beamsplitter is  $45^\circ$ , the two separated components have equal power so that the photocurrent noise due to the fluctuations of laser intensity will be completely subtracted. And the effective output of the balanced pair of photodetectors is zero until there is some difference in the powers of the two beams. With this setup, the laser intensity noise is cancelled and it is possible to detect small signal such as polarization rotation on a large DC background.

In order to have better noise subtraction, for example, in the case of using a noisy laser, the two photocurrents should be made as equal as possible. However, this requires many efforts and the noise subtraction is not perfect. To avoid this often manual task of balancing the two beam powers, we propose that an auto-balancing circuit can be employed. The circuit uses a low frequency feedback loop to maintain automatic DC balance between two photocurrents so as to allow easier detection of a small signal upon a large background.<sup>6</sup> It is reported that the noise floor can be improved to 3 dB above the shot noise of the signal beam.

To investigate the performance, we build the experiment to measure the air Faraday rotation with this auto-balanced technology. The Verdet constants of gases are smaller than liquids and solids by three orders of magnitude.<sup>1,7</sup> To measure the Faraday rotation in gases usually needs high magnetic field and long optical path length,<sup>8</sup> or cavity enhancement.<sup>9</sup> To induce the high magnetic field in Ingersoll's experiments, a large current of 11 A was used, so that a water flow was required to keep samples at constant temperature. Up to now, the best angular sensitivity for measuring the air Faraday rotation is  $2.93 \times 10^{-10} \text{ rad Hz}^{-1/2}$ , obtained by Jacob *et al.* with a cavity enhancement scheme. In their experiment, they used a 0.4 mW He-Ne laser at 633 nm as the light source and an optical cavity with a finesse of 7000 to enhance the signal. The shot noise limit of their experiment is  $\sim 7 \times 10^{-11} \text{ rad Hz}^{-1/2}$ .

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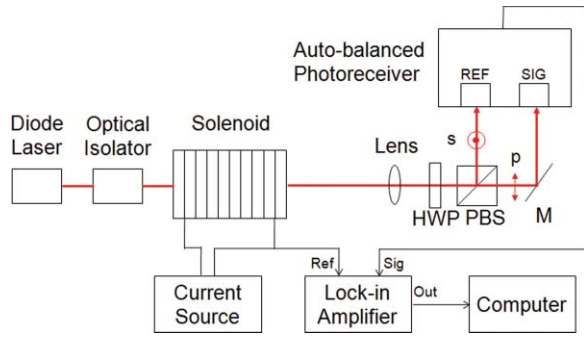


FIG. 1. (Color online) The experimental setup. HWP: half waveplate; PBS: polarizing beamsplitter; M: mirror.

Recently, an air Faraday rotation measurement using the two-frequency paired polarized interferometer by Lin *et al.* has demonstrated a sensitivity of  $2.7 \times 10^{-7} \text{ rad Hz}^{-1/2}$  in a single-pass configuration.<sup>10</sup>

In this report, the measurement of the tiny Faraday rotation of air with the auto-balanced photodetection is presented. The experimental setup, operation principle, and results are described. In addition to the performance of angular sensitivity, we will discuss the possibility of further improvement and potential applications.

## II. EXPERIMENTAL SETUP AND RESULTS

The experimental setup is depicted in Fig. 1. The light source is a semiconductor laser (MINI635-05, Blue Sky Research) delivering 4.3 mW linearly polarized output at 634.8 nm. An optical isolator is employed to prevent the optical feedback effect, and also reduce the power to meet the rating of the auto-balanced photoreceiver. The framework of the solenoid is made of Plexiglas. The solenoid is a 12 cm long six-layer coil with inner diameter 1.5 cm and outer diameter 3.0 cm, which can produce a longitudinal magnetic induction 82 Gauss/A. In our experiment, all optics is at least 50 cm away from the solenoid to eliminate the interference of their Faraday effects. The current source is a function generator (DS345, Stanford Research System) which can perform amplitude modulation at different depth. A resistor (not shown) of 1.02 ohm is connected in series with the solenoid to monitor the solenoid current using an oscilloscope and a capacitor (not shown) is also connected in series to form a  $LC$  circuit so that a sufficiently large current can be generated in the solenoid at the  $LC$  resonant frequency, 10 kHz. This current was 0.159 A and the induced magnetic field was 13.07 Gauss. There was no heating problem in the solenoid for long time operation at an ambient temperature of 22 °C. The auto-balanced photoreceiver Nirvana (2007, New Focus) is used in this work. The lens focuses the s- and p-polarization beams onto the signal (SIG) and reference (REF) detectors of the auto-balanced photoreceiver, respectively. The half waveplate (HWP) is used to setup the beam polarization to get a proper power splitting ratio for best SNR (see below for the details).

The auto-balanced photoreceiver output was sent to a lock-in amplifier (SR830, Stanford Research System), using a time constant of 300 ms with a slope of 6 dB so that the equiv-

alent noise bandwidth (ENBW) is equal to 5/6 Hz. The output of the lock-in amplifier was acquired by a personal computer every 40 ms and continued for 100 s. The SNR was the average value divided by the standard deviation.

We adopted the photoreceiver Nirvana (2007, New Focus) to perform the photodetection. The photoreceiver can carry out automatically balanced subtraction with a negative feedback loop.<sup>6,11</sup> The cutoff frequency of the negative feedback loop gives the maximum speed of the auto-balancing effect, and is determined by the loop bandwidth knob setting, signal photocurrent, and power splitting ratio. We must set the cutoff frequency below the modulation frequency of interest to make the signal to appear in the output. And in general, the photocurrents ratio of the two detectors  $I_{\text{REF}}/I_{\text{SIG}}$ , should be close to 2 for the best noise cancellation. But as stated below, the Nirvana's auto-balanced output is larger when the  $I_{\text{SIG}}$  is larger. So we set the ratio  $I_{\text{REF}}/I_{\text{SIG}}$  slightly less than 2, which is not a predetermined, exact number. As a result, the azimuth of the half waveplate has tolerance in a range.

The original beam polarization is assumed at an angle  $\theta$  with respect to the plane of incidence. When this polarization angle  $\theta$  is changed by adding a variation of  $\Delta\theta_0 \cos \omega t$ , where the rate  $\omega$  is higher than the cutoff frequency, the Nirvana's auto-balanced output voltage can be derived according to its manual in the following:

$$\Delta V_{\text{AB}} = -G (V_{\text{SIG}} \sin 2\theta) (\Delta\theta_0 \cos \omega t), \quad (1)$$

where  $V_{\text{SIG}}$  is the output voltage of the SIG detector at the balanced mode, and  $G$  is the gain factor of the auto-balanced output, which is

$$G = \frac{20}{\sin^2 2\theta}. \quad (2)$$

That is,

$$\Delta V_{\text{AB}} = -20 \left( \frac{V_{\text{SIG}}}{\sin 2\theta} \right) (\Delta\theta_0 \cos \omega t). \quad (3)$$

This output will be larger when  $V_{\text{SIG}}$  is larger or equivalently the ratio  $I_{\text{REF}}/I_{\text{SIG}}$  is smaller. And  $\Delta\theta_0$  is

$$\Delta\theta_0 = - \left( \frac{\sin 2\theta}{20V_{\text{SIG}}} \right) \Delta V_{\text{AB}0}, \quad (4)$$

where  $\Delta V_{\text{AB}0}$  is the amplitude of  $\Delta V_{\text{AB}}$ .

If  $\Delta\theta_0$  is caused by the Faraday effect with the light beam passing through a material of thickness  $d$  situated in a magnetic induction of  $B$ , the Verdet constant  $V$  of the material is

$$V = \frac{\Delta\theta_0}{Bd}. \quad (5)$$

In the experiment, we tuned the polarization angle  $\theta$  to set the power ratio  $I_{\text{REF}}/I_{\text{SIG}}$  to be around 2 but smaller, instead of setting  $\theta$  at 45° and simultaneously attenuating  $I_{\text{SIG}}$  by a factor of 2. The former procedure is more straightforward and can result in higher SNR. The s-polarization beam emerging from the PBS was 0.81 mW, and p-polarization beam was 0.47 mW. Then the angle  $\theta$  was:  $\theta = \tan^{-1} \sqrt{0.81/0.47} = 52.70^\circ$ . In this arrangement, the output voltages of the two detectors REF and SIG were  $V_{\text{REF}} = -7.79 \text{ V}$ , and  $V_{\text{SIG}} = 4.40 \text{ V}$ , which

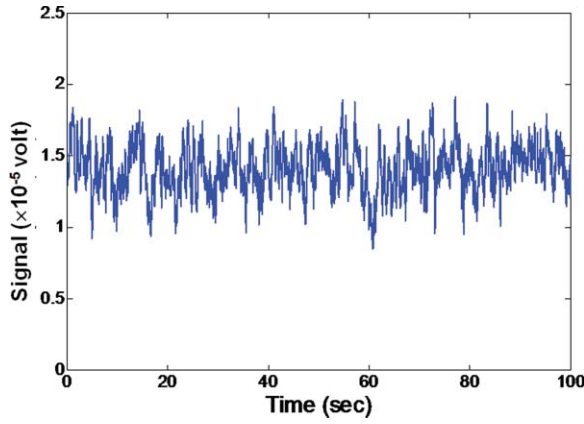


FIG. 2. (Color online) The output of the lock-in amplifier when the coil current was modulated at 10 kHz with an amplitude of 0.159 A.

were read at the photoreceiver's balanced mode by blocking the other beam separately.

The lock-in output for is shown in Fig. 2. Its average value is  $1.41 \times 10^{-5}$  V and SNR is 8.3. The measured phase angle of lock-in amplifier was  $-156.50^\circ$ .

$\Delta\theta$  is then computed as follows:

$$\Delta\theta = \left( \frac{\sin 2\theta}{20V_{\text{SIG}}} \right) \left( 1.41 \times 10^{-5} \times \sqrt{2} \right) = 2.18 \times 10^{-7} (\text{rad}). \quad (6)$$

And, the angular sensitivity of our measurement can be estimated to be

$$\left( 2.18 \times 10^{-7} \times \sqrt{1.2} \right) \times \frac{1}{8} \cong 2.99 \times 10^{-8} (\text{radHz}^{-1/2}). \quad (7)$$

At last, the derived Verdet constant of air is

$$V_{\text{air}} = \frac{2.18 \times 10^{-7}}{13.07 \times 12} = 1.39 \times 10^{-9} (\text{radG}^{-1} \text{cm}^{-1}). \quad (8)$$

By comparing the measured phase angle  $-156.50^\circ$  with those measured for a fused quartz glass of positive Verdet constant and a Faraday rotator glass MR3-2 of negative one from Xi'an Aofa Optoelectronics Technology Inc., it is convinced that the Verdet constant of air is positive and the value is around  $+1.39 \times 10^{-9} \text{ rad G}^{-1} \text{ cm}^{-1}$  at 634.8 nm.

As we know, there possibly exist the problems of baseline drifts, amplifier instabilities, and flicker noise in electronic circuits.<sup>12</sup> We used a double current modulation scheme to check if the problems exist in the measurement. The experimental setup for the double current modulation is depicted in Fig. 3. In this double modulation scheme, the 10 kHz sinusoidal signal from the function generator DS345 was modulated by a 6 Hz square wave such that its output was a sinusoidal signal multiplied by a square function of unit amplitudes +1 and -1. Equivalently the phase of the sinusoidal signal was changed by  $180^\circ$  every 1/12 s. The amplitude of solenoid current was 0.159 A as before. Both lock-in amplifiers are SR830 of Stanford Research System. The time constant of the first lock-in amplifier was set to 1 ms with a slope of 6 dB, and the sensitivity 1 mV so that its output could close to a square wave of 6 Hz. Then this output was sent to a second lock-in amplifier, which had the same time constant set-

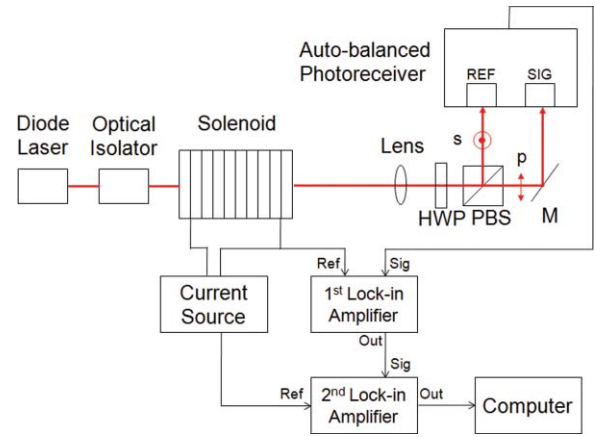


FIG. 3. (Color online) The experimental setup for double current modulation. HWP: half waveplate; PBS: polarizing beamsplitter; M: mirror

ting as that in the single modulation scheme. Similarly the second lock-in amplifier output was acquired by a personal computer every 40 ms for 100 s.

The lock-in amplifier SR830 can detect the first harmonic component of input signal, and output the root-mean-square value of this component. For a square wave input, its output yields 0.90 of the amplitude of the square wave. With these related parameters, the signal value  $\Delta V_{\text{AB0}}$  can be found.

The experimental result for the double current modulation is shown in Fig. 4. Its average value was 0.127 V and SNR was 8.4. Therefore the amplitude  $\Delta V_{\text{AB0}}$  of the signal square wave is

$$(1.27 \times 10^{-1}) \times (1 \times 10^{-4}) \times \frac{1}{0.9} \cong 1.41 \times 10^{-5} (\text{V}). \quad (9)$$

The value is the same as the result of single modulation scheme. Because the SNR is also almost the same, the angular sensitivity is  $2.99 \times 10^{-8} \text{ rad Hz}^{-1/2}$  too.

### III. DISCUSSIONS AND CONCLUSIONS

In our experiment,  $V_{\text{air}}$  is not in good agreement with the value  $1.9 \times 10^{-9} \text{ rad G}^{-1} \text{ cm}^{-1}$  obtained by Jacob *et al.*<sup>9</sup> This

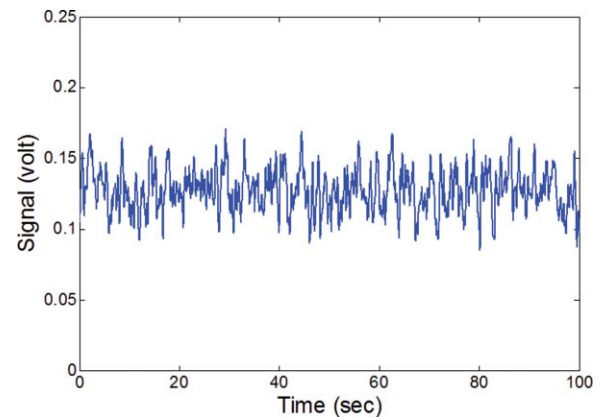


FIG. 4. (Color online) The output of the second lock-in amplifier in the double current modulation scheme.



maybe due to different ambient air conditions in the measurements. Our result closes to the value  $1.3 \times 10^{-9} \text{ rad G}^{-1} \text{ cm}^{-1}$  obtained by Lin *et al.* in a single-pass configuration using the two-frequency paired polarized interferometer.<sup>10</sup> In our experiment the total optical power is 1.28 mW, and the angular sensitivity is only 2.7 times the shot noise limit. In addition, the results of the double current modulation scheme were the same as the single modulation. This indicates that there was no zero drift and amplifier instability in our measurement.

Following the analysis in Ref. 13, it is possible to build up a Jones matrix model to describe the vectorial transmission of the Fabry-Perot interferometer. With this model, the Faraday rotation can be proved to be amplified in an optical resonator enclosing the solenoid. The result is that the Faraday rotation signal is multiplied by the factor  $2K\mathfrak{S}/\pi$  due to the optical resonator, where  $\mathfrak{S}$  is the Finesse of the resonator and  $K$  is the cavity transmission at resonance. That is, the angular sensitivity could be advanced by a factor of about  $2K\mathfrak{S}/\pi$ . For example, Jacob *et al.* used a resonator of finesse 7000 in their work.<sup>9</sup> By using a similar optical cavity and assuming  $K \cong 0.1$ , our sensitivity could be improved to about  $6.70 \times 10^{-11} \text{ rad Hz}^{-1/2}$ . This estimative value is 4 times better than the best obtained by Jacob *et al.*, although the laser power is only 3 times larger.

Recently organic materials have been investigated for efficient Faraday rotation because of their advantages in processing and application.<sup>3</sup> This sensitive polarimetry can also be used to characterize the magneto-optical properties of organic or polymeric materials, even to the single molecule level. This should be helpful for analyzing the mechanism of magneto-optic activity in these compositions and further development of organic materials with high Verdet constant.

A large Verdet constant material will transduce a small AC magnetic field into a larger alternating rotation of beam polarization. The polarimetry described in this paper could be easily implemented in this optical magnetometer. For this application, instead of lock-in amplifier, we can use fast Fourier

transformer to sense the frequency and magnitude of the magnetic field.

In summary, with the auto-balanced photodetection technique, the air Faraday rotation in a weak AC magnetic field was explored. The angular sensitivity reaches  $2.99 \times 10^{-8} \text{ rad Hz}^{-1/2}$ , which is only 2.7 times the shot noise limit. The Verdet constant of air obtained is  $+1.39 \times 10^{-9} \text{ rad G}^{-1} \text{ cm}^{-1}$  at 634.8 nm. In the experiments, the signal value and signal-to-noise ratio of the single modulation scheme is almost the same as the double current modulation. We expect that the sensitivity can be further improved after using a cavity enhancement configuration. Due to a flexibility in angular setup, this highly sensitive polarimetry should be also useful to incorporate with some magneto-optic applications.

## ACKNOWLEDGMENTS

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